



Mathematics In Context: A Lesson For Teaching Geometric Sequences In High School

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Article received: 07 Desember 2024, Review process: 17 Desember 2024,

Article Accepted: 22 Januari 2025, Article published: 31 Desember 2024

ABSTRACT

Learning design is an important framework in the educational process designed to achieve learning objectives effectively and efficiently. This research aims to analyze the teaching of Geometric Rows material in Class X SMA (Senior High School) using the Problem base Learning approach. This research uses design research method. The results of this study reveal that design research with a preliminary design stage succeeded in developing an "Iceberg" and HLT-based learning trajectory on geometric row material that was effective in facilitating students' concept understanding gradually. This learning trajectory is designed through stages ranging from real contexts to abstract models, so that students can understand the patterns and formulas of geometric rows in depth. At the contextual level, the introduction of real problems helps students relate mathematical concepts to everyday life, increasing learning motivation. At the concrete model level, students are able to visualize the geometric sequence pattern through manipulation of objects and tools, so that they understand the relationship between terms intuitively. Furthermore, at the abstract model level, students are not only able to use the geometric sequence formula correctly, but also understand the conceptual basis behind it. Finally, at the generalization level, students are able to solve various application problems, showing critical thinking skills and a deeper understanding of the concepts.

Keywords: Learning Design, Problem Solving, Learning Strategies

ABSTRAK

Desain pembelajaran merupakan kerangka penting dalam proses Pendidikan yang dirancang untuk mencapai tujuan pembelajaran secara efektif dan efisien. Penelitian ini bertujuan untuk menganalisis pengajaran materi Barisan Geometri di Kelas X SMA (Sekolah Menengah Atas) dengan menggunakan pendekatan Pemecahan Masalah (Problem base Learning). Penelitian ini menggunakan metode penelitian desain atau design research. Hasil penelitian ini mengungkapkan bahwa penelitian desain dengan tahap preliminary design berhasil mengembangkan lintasan belajar berbasis "Iceberg" dan HLT pada materi barisan geometri yang efektif dalam memfasilitasi pemahaman konsep siswa secara bertahap. Lintasan belajar ini dirancang melalui tahapan melalui tahapan mulai dari konteks nyata hingga model abstrak, sehingga siswa dapat memahami pola dan rumus barisan geometri secara mendalam. Pada level kontekstual, pengenalan masalah nyata membantu siswa

mengaitkan konsep matematika dengan kehidupan sehari-hari, meningkatkan motivasi belajar. Pada level model konkret, siswa mampu memvisualisasikan pola barisan geometri melalui manipulasi objek dan alat bantu, sehingga mereka memahami hubungan antar suku secara intuitif. Selanjutnya, pada level model abstrak, siswa tidak hanya mampu menggunakan rumus barisan geometri dengan benar, tetapi juga memahami dasar konseptual di baliknya. Terakhir, pada level generalisasi, siswa mampu menyelesaikan berbagai soal aplikasi, menunjukkan kemampuan berpikir kritis dan pemahaman yang lebih mendalam tentang konsep.

Kata Kunci: *Desain Pembelajaran, Pemecahan Masalah, Strategi Pembelajaran*

INTRODUCTION

Number ranks and patterns are very important material in mathematics, especially the material of geometric ranks which is very much used in its application in other materials involving exponential growth or decay, such as population growth models, compound interest calculations, or radioactive decay predictions. In addition to the field of mathematics, the geometric row is also indispensable in calculating the number of mycoorganism division in the field of biology, in the field of physics can also be used to determine the length of the trajectory of the ball dropped from a certain height until it stops, therefore the material of the geometric row is very important to be mastered by high school class X students because of its benefits in everyday life and in other subject matter.

The reality shown by students in learning, many students who experience confusion in solving problems related to geometric row material, especially if faced with problems - more complex problems. The lack of understanding of students can be caused by the inappropriate approach taken by educators, namely teachers in delivering learning. Most students do not understand the concept of geometric rows when solving the problems they face because students tend to memorize the solution method, so that when given a problem that is much different from the example given, students experience confusion in solving the given geometric row problem. As research conducted by Hartati (2021), students do not understand the concept of geometric and arithmetic rows and series, students have difficulty in the process of working, especially when calculating it can be seen from student errors in calculating exponential multiplication, students have difficulty in distinguishing between arithmetic and geometric row problems.

In line with this research conducted by Haryomurti, Prabawanto & Jupri, (2020), learning difficulties or obstacles experienced by students consist of ontogenic constraints, didactical constraints, and epistemological constraints. Ontogenic constraints are student errors in solving related to the material, for example confused in understanding the meaning of the problem, writing the wrong formula, not knowing how to find the value of n , unable to determine the ratio of the geometric row, not entering the value of U_n correctly and not being able to understand using the formula for the n th term of the geometric row correctly. Didactical constraints are errors in teaching, for example, teachers are not good at providing prerequisite knowledge, students also feel that using manual methods is easier than using formulas in solving number and number line problems, and

students feel that teachers are not clear in conveying number and geometric line material. The epistemological obstacles are errors regarding basic knowledge such as the rules of arithmetic operations, for example, incorrectly using the rules of addition and multiplication, procedural errors, namely in applying the distributive property and errors in transferring segments, namely multiplication into division.

Although traditional learning methods are widely used, previous research shows that more interactive and problem-based approaches can improve and engage students, resulting in better student understanding in concept mastery. Therefore, there is a significant gap between the current teaching conditions that tend to be monological and ideal conditions where students are actively involved in the learning process through problem solving activities. According to Sanjaya (2006), mathematics learning that is too teacher-centered tends to make students passive so that understanding of concepts becomes less than optimal. Therefore, a learning method that can actively involve students in the process of thinking and problem solving is needed.

One of the relevant approaches is the problem solving approach. According to Polya (1973), problem solving is a thinking process that involves students in understanding, formulating, and solving problems. This method emphasizes active student involvement in the learning process, so that students not only memorize formulas, but also understand the concepts and applications. With problem solving, students are invited to think critically, creatively, and systematically in solving mathematical problems. In addition, Arends (2012) states that the problem solving approach can help students develop higher order thinking skills because it involves analysis, evaluation, and synthesis. In the context of learning geometry rows, this method allows students to understand the patterns and relationships between row terms more deeply, while training them to apply these concepts in real situations.

The results of previous research show that the problem solving method can improve students' concept understanding in learning mathematics. According to Hudojo (2005), the problem solving approach provides students with meaningful learning experiences because they are invited to identify problems, formulate solutions, and reflect on the results. This is in line with the purpose of learning mathematics, which is to develop students' ability to solve problems in a logical, critical, and creative way (Permendikbud No. 37 of 2018 concerning the 2013 curriculum).

In learning geometry rows in class X SMA, the application of problem solving method is expected to overcome students' difficulties in understanding the concept, improve critical thinking skills, and foster learning motivation. Therefore, this article is focused on the problem solving method in teaching geometry row material, with the aim of improving students' concept understanding while building students' ability to solve mathematical problems effectively.

METHODS

This research uses design research method or design research is a systematic approach that aims to develop effective learning designs, including learning

trajectory of geometry based on problem solving approach. This approach is oriented towards the development of local theory (local instructional theory) and solving practical problems in the classroom. According to Gravemeijer and Cobb (2006), design research not only solves practical solutions, but also expands theoretical knowledge related to learning. In developing a learning trajectory for geometry rows, this process involves three main stages, namely preliminary design, testing and revision (prototyping phase), and implementation and generalization (assessment phase). However, this research is only limited to the preliminary design stage. Design research with a preliminary design stage is a very important first step to develop a systematic and effective learning trajectory. In the context of developing an "Iceberg"-based learning trajectory on geometric sequence material, this stage focuses on analyzing learning needs, planning objectives, and preparing the initial learning design.

RESULTS AND DISCUSSION

In this preliminary design stage, the researcher developed an "iceberg" learning trajectory on geometric row material. The preparation of the Hypothetical Learning Trajectory (HLT) was preceded by an analysis of curriculum literature to ensure that the designed learning is in accordance with the applicable mathematics curriculum for grade X high school students as subjects in learning activities. The analysis includes determining teaching materials, learning objectives, and learning indicators. After determining the learning objectives and indicators, the next activity is to analyze the literature related to geometric row material. In this literature analysis, it was found that students still have confusion in solving problems related to geometric row material. The results of the needs analysis show that students often have difficulty understanding geometric sequence patterns, especially in recognizing fixed ratios between terms, using the n th term formula. This difficulty is generally caused by students' lack of experience in connecting abstract concepts with real contexts. In addition, the analysis also found that geometric sequence material is often taught directly with formulas without giving students the opportunity to explore patterns independently.

Before making the trajectory of geometric row material, students already understand the basic material of number patterns and arithmetic rows, the following concept map of Row and Series material.

After the "Iceberg" based learning trajectory is designed and implemented in small groups of grade X students, by presenting from real concepts, (Real world situation), proceeding to the concept of building understanding from real objects (model of situation), to find the number pattern of the geometric row and find the formula of each term from the image presented (Formal model of knowledge building), until finally to the abstract understanding of finding the formula of the n th term of the geometric row (abstract formal), This can be seen from Figure 1.

The results show a gradual increase in concept understanding. At the contextual level, students are able to identify patterns in real problems, such as the number of matchsticks arranged in a triangle and following the pattern of geometric

rows, the number of balls arranged in a pyramid or population growth with compound interest. This activity helps students understand the concept of geometric rows in everyday life. Furthermore, at the concrete model level, students successfully visualize the geometric row pattern using tools such as paper folded to form a geometric row pattern, using coins or diagrams, so that learners can understand the pattern of relationships between terms. At the abstract model level, students show a better understanding of the formula for the n th term of the geometric row, students are not only able to apply the formula, but also understand how the formula is related to the pattern they have previously explored, for example students are able to explain that $n-1$ in the u_n formula describes the change in the ratio between terms from the beginning to the n th term.

At the generalization level, students successfully solve complex problems, such as calculating the total population based on the annual growth rate that forms a geometric sequence, or determining the number of bacterial divisions in a certain period of time. The increase in student understanding at each level shows that the "Iceberg" based learning trajectory is effective in facilitating the learning of geometric rows. This approach allows students to understand concepts gradually, starting from familiar real contexts to abstract mathematical models. This is in line with Freudenthal's (1991) opinion in the theory of Realistic Mathematics Education (RME), which states that mathematics learning should start from situations that are meaningful to students to support deep understanding.

Success at the contextual level indicates that the introduction of real problems can attract students' interest and provide motivation to learn. This finding supports Gravemeijer's (1994) view, which states that contextual problems provide a foundation for building conceptual understanding. Furthermore, student success at the concrete model level indicates that visualization and manipulation of concrete objects can help students understand geometric row patterns. The use of tools such as tables, diagrams, or matchsticks helps students connect visual patterns with mathematical patterns.

At the abstract model level, students are able to understand and apply the formulas of geometric rows because students have had exploratory experiences in the previous stage. This shows the importance of the relationship between concrete and abstract models in learning, as stated by Bakker (2018), that the transition from concrete to abstract is the core of an effective learning trajectory. Students' success at the generalization level also shows that this learning trajectory not only helps students understand the concept, but also improves their ability to apply the concept of geometric rows in various contexts.

However, this study also found that some students needed more time at the concrete model level to understand the relationship between terms before moving on to the abstract model level. This shows that students' needs are different, and teachers need to provide guidance according to each student's ability. In addition, the implementation of the "Iceberg"-based learning trajectory requires more time compared to the traditional approach, but this additional time pays off with a positive impact on students' learning deeper understanding.

Thus the “Iceberg” based learning trajectory can be an effective approach to overcome students' difficulties in understanding geometry rows. This approach allows students to build conceptual understanding gradually, improve critical thinking skills, and strengthen students' ability to apply concepts in real life. This research is expected to be the basis for the development of more innovative and contextualized mathematics learning.

CONCLUSION

The conclusion of this research can be presented that the design research with the preliminary design stage succeeded in developing a learning trajectory based on “Iceberg” and HLT on geometric row material that is effective in facilitating students' concept understanding gradually. This learning trajectory is designed through stages ranging from real context to abstract models, so that students can understand the patterns and formulas of geometric rows in depth. At the contextual level, the introduction of real problems helps students relate mathematical concepts to everyday life, increasing learning motivation. At the concrete model level, students are able to visualize the geometric sequence pattern through manipulation of objects and tools, so that they understand the relationship between terms intuitively. Furthermore, at the abstract model level, students are not only able to use the geometric sequence formula correctly, but also understand the conceptual basis behind it. Finally, at the generalization level, students are able to solve various application problems, showing critical thinking skills and a deeper understanding of the concepts.

This “Iceberg” based approach is proven to support a more meaningful learning process, in accordance with the theory of Realistic mathematics Education by Freudenthal (1991), which emphasizes the importance of contextual and gradual learning in mathematics. The results of this study also show that students more easily understand the concept of geometric rows because the transition from the concrete stage to the abstract stage is done systematically. Although its implementation takes longer, this learning trajectory has a positive impact on students' understanding and their ability to apply concepts in various situations. Therefore, the Iceberg-based learning trajectory can be an innovative and effective alternative approach in learning mathematics, especially for geometric rows.

LIST OF REFERENCES

- Jonassen, D. H, “Supporting Problem Solving in PBL,” *Interdisciplinary Journal of Problem-Based Learning*, vol. 5, no. 2; pp. 95-119, 2011
- Widjajanti, D. B, “Pengembangan Lintasan Belajar Berbasis Realistic Mathematics Education (RME) untuk materi barisan dan deret,” *Jurnal Pendidikan Matematika*, vol. 5, no. 1: pp. 10 - 20, 2011.
- Suryadi, D, “Pendekatan Hypothetical Learning Trajectory (HLT) dalam pembelajaran matematika,” *infinity Jurnal*, vo. 2, no. 2, pp. 161 - 172, 2013.
- Eric, “Education Resources Information Center,” <https://eric.ed.gov>
- Springer Link, <https://link.springer.com>

Google Scholar, <https://scholar.google.com>

Bakker, A. *Design Research In Education : A Practical Guide for early Career Researchers*. Routledge, 2018.

Freudenthal, H. *Revisiting Mathematics Education: China Lectures*. Kluwer Academic Publisher, 1991.

Suryadi, D, "Pendekatan Hypothetical Learning Trajectory (HLT) dalam pembelajaran matematika". *Infinity Journal*, 2(2), 2013, pp. 161 - 172.

Van Den Heuvel-Panhuizen, M, "The Didactical Use of Models In Realistic Mathematics Education: An Example from a Longitudinal Trajectory on Measurement and Geometry". *Educational Studies In Mathematics*, 2003, pp. 9 - 35.

Gravemeijer, M.A, *Developping Realistic Mathematics Education*. Utrecht: Freudenthal Institute, 1994.

R. L. Myer, "Parametric oscillators and nonlinear materials," in *Nonlinear Optics*, vol. 4, P. G. Harper and B. S. Wherret, Eds., San Francisco, CA, USA: Academic, 1977, pp. 47-160.